

# Colorado Measures of Academic Success



# Grade 8 Mathematics

## Answer Key with Scoring Rubrics

Practice Resource for Students

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## ITEM INFORMATION

Colorado Academic Standard (CAS) Evidence Outcome

Describes the evidence that demonstrates that a student is meeting the grade level expectation at a mastery level.

Evidence Statement

Describes the knowledge or skills that an assessment item/task elicits from students. Full descriptions of Evidence Statements and their alignment to the Colorado Academic Standards are located at [http://cde.state.co.us/assessment/cmas\\_testdesign](http://cde.state.co.us/assessment/cmas_testdesign).

Subclaim

The reporting category of the associated CAS.

- Mathematics
  - Subclaim A – Major Content
  - Subclaim B – Supporting Content
  - Subclaim C – Expressing Mathematical Reasoning
  - Subclaim D – Modeling and Application

## ITEM TYPES

Items are questions that appear on the assessments. They are presented in three different ways.

**Selected Response (Multiple Choice, Multiple Response, and Fill in the Blank):**

For multiple choice and multiple response items, students select a correct answer out of provided choices. For fill in the blank items, students type/write their answer in a blank box.

**Technology-Enhanced (Bar Graph, Drag and Drop, Inline Choice, Hot Spot, and Match Table Grid):**

Students show their answer using technology, such as by creating a bar graph using a template provided by the online testing system or on the paper-based test. Drag and drop items require students to drag answer choices into correct answer bays (draw lines or write corresponding letters for paper-based testing). Inline choice items require students to select their answer from a drop-down menu (circle answer from a list of choices for paper-based testing) to complete a sentence or sentences. Hot spot items require students to select the correct response from its location in an image (write corresponding letters or circle answer for paper-based testing). Match table grid items require students to check checkboxes in cells to indicate a match between the column and row labels.

**Constructed Response:**

Students construct an open-ended response.

## STUDENT PERFORMANCE

**P Value – Selected Response Only**

The P value represents the percentage of students who answered each selected response question correctly. For example, if the P value associated with a question is 0.64, then 64% of students responded to the question with the correct answer.

**Score Point Distribution – Constructed Response Only**

The score point distribution provides the percentage of students who scored at each possible score point for constructed response questions.

In addition to score point distribution, the scoring guide, scoring rubric, and sample student responses at each score point are provided for constructed response items.

## ANSWER KEY: ITEM SET 1

### Item Set 1 - Question 1 (Multiple Select)

Which input-output table represents a nonlinear function?

Select **each** nonlinear function.

A.

Input ( $x$ )	Output ( $y$ )
2	3
3	7
4	11

B.

Input ( $x$ )	Output ( $y$ )
2	4
4	8
6	12

C.

Input ( $x$ )	Output ( $y$ )
2	2
4	14
6	34

D.

Input ( $x$ )	Output ( $y$ )
2	5
3	10
4	17

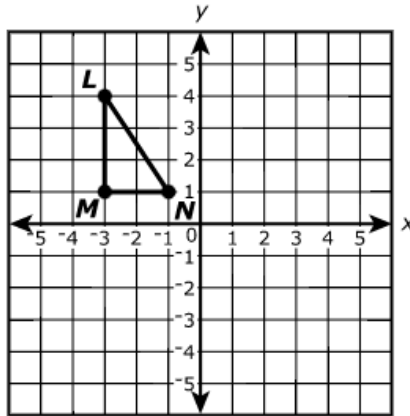
E.

Input ( $x$ )	Output ( $y$ )
2	2
4	3
6	4

Item Information		
Answer:	C, D	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.F.A.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.
Evidence Statement:	8.F.3-2	Give examples of functions that are not linear and prove that they are not linear. i) Tasks have "thin context" or no context. ii) Tasks require students to demonstrate understanding of function nonlinearity, for example by recognizing or producing equations that do not define linear functions, or by recognizing or producing pairs of points that belong to the graph of the function yet do not lie on a straight line. iii) Tasks do not require students to produce a proof; for that aspect of standard 8.F.3, see 8.C.3.1. iv) Tasks involving symbolic representations are limited to polynomial functions i.e., $y = 3x^2 + 2$ .
Subclaim:	A – Major Content	The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.275	

Item Set 1 – Question 2 (Selected Response)

Triangle  $LMN$  is shown on the coordinate plane.



Triangle  $L'M'N'$  is the image of triangle  $LMN$  after a reflection.

Which statement is true about the line segments in the two triangles?

- A.  $\overline{NL}$  is the same length as  $\overline{M'N'}$ .
- B.  $\overline{NL}$  is the same length as  $\overline{N'L'}$ .
- C.  $\overline{LM}$  is shorter than  $\overline{L'M'}$ .
- D.  $\overline{MN}$  is longer than  $\overline{L'M'}$ .

Item Information		
Answer:	B	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.G.A.1.a	Lines are taken to lines, and line segments to line segments of the same length.
Evidence Statement:	8.G.1a	Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. i) Tasks do not have a context.
Subclaim:	A – Major Content	The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.787	

Item Set 1 – Question 3 (Fill in the Blank)

An ant's mass is  $5 \times 10^{-3}$  grams. A bee's mass is  $1 \times 10^{-1}$  grams.

Based on this information, how many ants equal the mass of one bee?

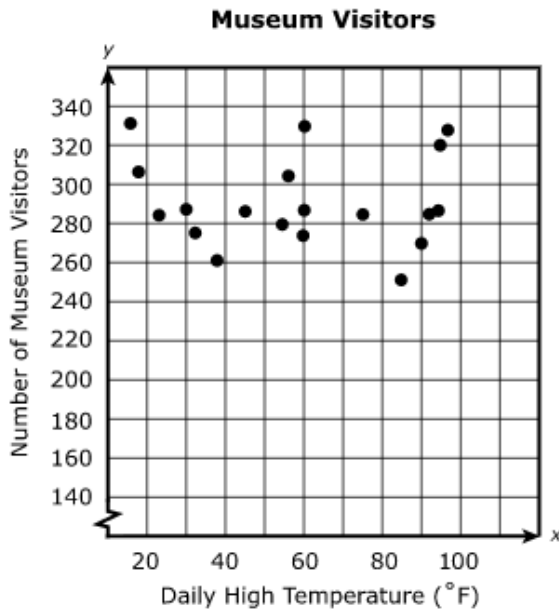
Enter your answer in the box.

20

Item Information		
Answer:	See Image	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times $10^8$ and the population of the world as 7 times $10^9$ , and determine that the world population is more than 20 times larger.
Evidence Statement:	8.EE.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as $3 \times 10^8$ and the population of the world as $7 \times 10^9$ , and determine that the world population is more than 20 times larger.
Subclaim:	A – Major Content	The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.15	

Item Set 1 – Question 4 (Selected Response)

The scatter plot shows the number of museum visitors compared to the daily high temperature, in degrees Fahrenheit. The number of museum visitors is represented by  $y$ , and the daily high temperature is represented by  $x$ .



Which phrase **best** describes the association between the number of museum visitors and the daily high temperature?

- A. negative linear association
- B. positive linear association
- C. nonlinear association
- D. no association

Item Information		
Answer:	D	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.SP.A.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
Evidence Statement:	8.SP.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. i) Tasks might have spreadsheet-like technology features, such as the ability to select data ranges for the two axes and have the scatter plot automatically generated.
Subclaim:	B - Supporting Content	The student solves problems involving the Additional and Supporting Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.333	



Item Set 1 – Question 5 (TEI Equation Editor)

Solve the system of equations.

$$4x + 5y = 20$$

$$3x + 10y = 20$$

Enter your answer in the space provided. Enter **only** your answer.

(4, 0.8)

Item Information		
Answer:	See Image	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.EE.C.8.b	Solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.
Evidence Statement:	8.EE.8b-1	Analyze and solve pairs of simultaneous linear equations. b. Solve systems of two linear equations in two variables algebraically.
Subclaim:	A – Major Content	The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.111	

Item Set 1 – Question 6 (Selected Response)

**Part A**

Triangle  $JKL$  and triangle  $MPN$  are similar.

Which single transformation maps  $\triangle JKL$  to  $\triangle MPN$ ?

- A. translation
- B. reflection
- C. rotation
- D. dilation

**Part B**

Which statement **best** describes the relationship between  $\triangle JKL$  and  $\triangle QRS$ ?

- A. The triangles are not similar because  $\triangle QRS$  cannot be mapped onto  $\triangle JKL$  by a sequence of transformations.
- B. The triangles are similar because  $\triangle QRS$  can be mapped onto  $\triangle JKL$  by a sequence of transformations.
- C. The triangles are similar because they have different perimeters.
- D. The triangles are not similar because they have different areas.

Item Information		
Answer:	Part A = B, Part B = B	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.G.A.4	Demonstrate that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
Evidence Statement:	8.G.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. i) Tasks do not have a context. ii) Tasks do not reference congruence (this relationship will be assessed in 8.C.3.2). iii) Tasks should not focus on coordinate Geometry; figures may be drawn in the coordinate plane, but do not include the use of coordinates. iv) Tasks should elicit student understanding of the connection between similarity and transformations i.e., tasks may provide two similar figures

		and require the description of a sequence of transformations that exhibits the similarity or tasks may require students to identify whether two figures are similar using a sequence of transformations. v) Tasks do not require students to indicate a specified scale factor. vi) Similarity should not be obtained through the proportionality of corresponding sides.
Subclaim:	A – Major Content	The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.733	

Item Set 1 – Question 7 (Selected Response)

Which decimal is equivalent to  $\frac{11}{15}$ ?

A.  $0.\overline{733}$

B. 0.73

C.  $0.7\overline{3}$

D. 0.733

Item Information		
Answer:	C	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.NS.A.1	Demonstrate informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number. Define irrational numbers as numbers that are not rational.
Evidence Statement:	8.NS.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number. i) Tasks do not have a context. ii) 50% of tasks require students to write a fraction $a/b$ as a repeating decimal by showing, filling in, or otherwise producing the steps of a long division $a \div b$ . iii) 50% of tasks require students to write a given repeating decimal as a fraction. iv) For tasks that involve writing a repeating decimal as a fraction, the given decimal should include no more than two repeating decimals without non-repeating digits after the decimal point. (i.e., 2.16666..., 0.23232323...).
Subclaim:	B - Supporting Content	The student solves problems involving the Additional and Supporting Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.627	

Item Set 1 – Question 8 (Selected Response)

Which expression is equivalent to  $4^5 \times 4^{-3}$ ?

A.  $4^8$

B.  $4^2$

C.  $4^{-2}$

D.  $4^{-8}$

Item Information		
Answer:	B	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$ .
Evidence Statement:	8.EE.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 1/3^3 = 1/27$ . i) Tasks do not have a context. ii) Tasks center on the properties and equivalence, not on simplification. For example, a task might ask a student to classify expressions according to whether or not they are equivalent to a given expression. iii) 50% of expressions should involve one property. iv) 30% of expressions should involve two properties. v) 20% of expressions should involve three properties. vi) Tasks should involve a single common base or a potential common base, such as a task that includes 3, 9 and 27.
Subclaim:	A – Major Content	The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.641	

Item Set 1 – Question 9 (Multiple Select)

The table shows a function composed of the given input and output values.

Function	
Input	Output
2	1
3	3
4.5	6
?	?

Which sets of values could be included in the function?

Select **all** possible sets of values.

- A. 

Input	Output
1	2
- B. 

Input	Output
2	3
- C. 

Input	Output
3	2
- D. 

Input	Output
-3	3
- E. 

Input	Output
-4.5	6

**Item Information**

Answer:	A, D, E	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.F.A.1	Define a function as a rule that assigns to each input exactly one output. Show that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required for Grade 8.)
Evidence Statement:	8.F.1-1	Understand that a function is a rule that assigns to each input exactly one output. i) Tasks do not involve the coordinate plane or the "vertical line test." ii) Tasks do not require knowledge of the concepts or terms domain and range. iii) 20% of functions in tasks are non-numerical, e.g., the input could be a person and the output could be his or her month of birth. iv) Tasks should involve clearly defined inputs and outputs.
Subclaim:	A – Major Content	The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.108	

Item Set 1 – Question 10 (TEI Hot Spot)

Select a point on the number line that **best** approximates the location of  $\sqrt{10}$ .

Select **one** place on the number line to plot the point.



Item Information		
Answer:	See Image	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi^2$ ). For example, by truncating the decimal expansion of $\sqrt{2}$ , show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.
Evidence Statement:	8.NS.2	Use rational approximations of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g. $\pi^2$ ). For example, by truncating the decimal expansion of $\sqrt{2}$ , show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. i) Tasks do not have a context.
Subclaim:	B - Supporting Content	The student solves problems involving the Additional and Supporting Content for her grade/course with connections to the Standards for Mathematical Practice.

Item Set 1, Calculator Section– Question 11 (Selected Response)

A randomly selected sample of middle school students were asked whether they have been on a field trip to a museum and whether they have participated in an after-school camp. The table shows the results of the survey.

**Survey Results for Middle School Students**

	Have Been on a Field Trip to a Museum	Have Not Been on a Field Trip to a Museum	Total
Have Participated in an After-School Camp	44	28	72
Have Not Participated in an After-School Camp	45	43	88
<b>Total</b>	<b>89</b>	<b>71</b>	<b>160</b>

Based on the data in the table, which statement is true about the middle school students?

- A. There are 28 students who have participated in an after-school camp but have not been on a field trip to a museum.
- B. There are 45 students who have participated in an after-school camp and have been on a field trip to a museum.
- C. A total of 43 students have not been on a field trip to a museum.
- D. A total of 89 students have participated in an after-school camp.

**Item Information**

Answer:	A	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.SP.A.4	Explain that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?
Evidence Statement:	8.SP.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores? ) One-third of tasks involve basic comprehension questions about a two-way table, such as “How many students who don’t have chores have a curfew?” ii) One-third of tasks involve computing marginal sums or marginal percentages. iii)



		One-third of tasks involve interpretation or patterns of association. iv) Tasks that require finding missing values within the categories are excluded. v) Tasks are limited to two categorical variables with two subcategories each. vi)The testing interface can provide students with a calculation aid of the specified kind for these tasks. vii) The joint frequencies must be provided either in a table or within the context. The marginal frequencies do not need to be provided.
Subclaim:	B - Supporting Content	The student solves problems involving the Additional and Supporting Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.625	

## ANSWER KEY: ITEM SET 2, CALCULATOR SECTION

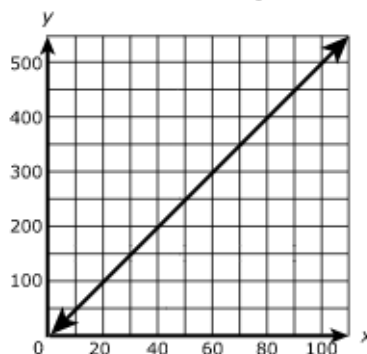
### Item Set 2 - Question 1 (TEI Inline Choice)

Two relationships are shown. The table shows the relationship between  $x$  and  $y$  in Relationship A. The graph shows a different relationship between  $x$  and  $y$  in Relationship B. The units for  $x$  and  $y$  are the same in each relationship.

**Relationship A**

$x$	$y$
3	18
6	36
9	54
12	72

**Relationship B**



Select from the drop-down menus to correctly complete the sentence.

The slope of the graph of Relationship A is  the slope of the graph of Relationship B because the unit rate of Relationship A is  the unit rate of Relationship B.

#### Item Information

Answer:	See Image	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
Evidence Statement:	8.EE.5-2	Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has a greater speed. i) Pool should contain tasks with and without contexts. ii) The testing interface can provide students with a calculation aid of the specified kind for these tasks.
Subclaim:	A – Major Content	The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.326	

Item Set 2 - Question 2 (Constructed Response)

A customer spends \$21.50 on cupcakes and muffins. The number of muffins purchased is 1 fewer than the number of cupcakes.

Each cupcake costs \$2, and each muffin costs \$1.25.

- Create a system of equations that relates  $c$ , the number of cupcakes, and  $m$ , the number of muffins, the customer purchased.
- Determine the total cost of the muffins the customer purchased. Show or explain your work.

Enter your equations, your answer, and your work or explanation in the space provided.

Item Information		
Answer:	See Sample Student Responses and Scoring Rubric	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.EE.C.8.c	Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
Evidence Statement:	8.D.1	Solve multi-step contextual word problems with degree of difficulty appropriate to grade 8, requiring application of knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements. i) Tasks may have scaffolding if necessary, in order to yield a degree of difficulty appropriate to grade 8.
Subclaim:	D – Modeling and Application	The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course (or for more complex problems, knowledge and skills articulated in the standards for previous grades/courses), engaging particularly in the Modeling practice, and where helpful making sense of problems and persevering to solve them (MP. 1), reasoning abstractly and quantitatively (MP. 2), using appropriate tools strategically (MP.5), looking for and making use of structure (MP.7), and/or looking for and expressing regularity in repeated reasoning (MP.8).
P Value:	0.125	

Scoring Rubric	
Points	Attributes
3	<p>Student response includes each of the following 3 elements.</p> <ul style="list-style-type: none"> <li>• <b>Computation component:</b> Correct total cost of the muffins the customer purchased, \$7.50</li> <li>• <b>Modeling component:</b> Valid work or explanation on how to determine the total cost of the muffins the customer purchased and/or valid work for solving their system of equations</li> <li>• <b>Modeling component:</b> Correct system of equations</li> </ul> <p><b>Note:</b> Student does not need to show or explain how they got their system of equations.</p>
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 3 elements.
0	Student response is incorrect or irrelevant.

<p>Sample Student Response:</p>	<p><b>Sample Solution 1:</b>  <math>2c + 1.25m = 21.50</math>  <math>m = c - 1</math>  <math>2c + 1.25(c - 1) = 21.50</math>  The equation needs to be solved to see how many cupcakes and muffins the customer purchased.  <math>2c + 1.25(c - 1) = 21.50</math>  <math>2c + 1.25c - 1.25 = 21.50</math>  <math>3.25c - 1.25 = 21.50</math>  Add 1.25 to each side and you get:  <math>3.25c = 22.75</math>  Divide each side by 3.25, you get 7 as your answer to cupcakes. Since muffins are one less than cupcakes you take one off, so it makes 7 cupcakes and 6 muffins. To find the total cost of muffins the customer purchased multiply 6 by the amount each muffin costs, \$1.25. The total cost of the muffins that the customer bought was \$7.50.</p>
<p>Annotation for Sample Student Response:</p>	<p><b>Solution 1, Score Point 3</b>  The response receives full credit. It includes each of the 3 required elements.</p> <p><b>Computation Component:</b></p> <ul style="list-style-type: none"> <li>• <b>Student Response:</b> The total cost of the muffins that the customer bought was \$7.50 <ul style="list-style-type: none"> <li>○ <b>Rationale for Score:</b> The correct total cost of the muffins that the customer purchased is provided (\$7.50).</li> </ul> </li> </ul> <p><b>Modeling Component:</b></p> <ul style="list-style-type: none"> <li>• <b>Student Response:</b> <math>2c + 1.25(c - 1) = 21.50</math>, <math>2c + 1.25c - 1.25 = 21.50</math>, <math>3.25c - 1.25 = 21.50</math>, add 1.25 to each side . . . <math>3.25c = 22.75</math>, divide each side by 3.25, you get 7 as your answer to cupcakes. Since muffins are one less than cupcakes you take one off, so it makes 7 cupcakes and 6 muffins . . . you multiply 6 by the amount each muffin costs, \$1.25 <ul style="list-style-type: none"> <li>○ <b>Rationale for Score:</b> The student provides valid work [<math>2c + 1.25(c - 1) = 21.50</math>, <math>2c + 1.25c - 1.25 = 21.50</math>, <math>3.25c - 1.25 = 21.50</math>, add 1.25 to each side . . . <math>3.25c = 22.75</math>, divide each side by 3.25, you get 7 as your answer to cupcakes] and explanation (Since muffins are one less than cupcakes you take one off, so it makes 7 cupcakes and 6 muffins . . . you multiply 6 by the amount each muffin costs, \$1.25) of how to determine the total cost of muffins.</li> </ul> </li> </ul> <p><b>Modeling Component:</b></p> <ul style="list-style-type: none"> <li>• <b>Student Response:</b> <math>2c + 1.25(c - 1) = 21.50</math>, <math>2c + 1.25c - 1.25 = 21.50</math> <ul style="list-style-type: none"> <li>○ <b>Rationale for score:</b> The student provides the correct system of equations (<math>2c + 1.25(c - 1) = 21.50</math>, <math>2c + 1.25c - 1.25 = 21.50</math>).</li> </ul> </li> </ul> <p><b>Note:</b> Sample student responses are not representative of all correct answers for an item and are only provided as a guide to assist teachers with scoring.</p>

Item Set 2 - Question 3 (TEI Fill in the Blank, Selected Response)

**Part A**

According to the model, how many ounces of ice cream are in the container before any scoops are sold?

Enter your answer in the box.

220

**Part B**

What does the slope of the equation for the linear model represent?

- A. the change in the number of scoops of ice cream sold per ounce of ice cream remaining in the container
- B. the change in the number of ounces of ice cream remaining in the container per scoop of ice cream sold
- C. the number of ounces of ice cream remaining in the container
- D. the number of ice cream scoops sold

**Part C**

According to the model, how many ounces of ice cream remain after selling 43 scoops?

Enter your answer in the box.

48

**Part D**

The graph of the linear model intersects the  $x$ -axis at  $(55, 0)$ .

What does this intersection point represent in this situation?

- A. There are 0 ounces of ice cream remaining in the container after selling 55 scoops.
- B. There are 55 ounces of ice cream remaining in the container after selling 0 scoops.
- C. There are 55 scoops of ice cream remaining in the container.
- D. There are 55 ounces in each scoop of ice cream sold.

Item Information		
Answer:	Part A = See Image, Part B = B, Part C = See Image, Part D = A	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
Evidence Statement:	8.SP.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. i) Tasks are word problems based on bivariate measurement data that require students to use the equation of a linear model. ii) The testing interface can provide students with a calculation aid of the specified kind for these tasks.
Subclaim:	B - Supporting Content	The student solves problems involving the Additional and Supporting Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.426	

Item Set 2 - Question 4 (TEI Equation Editor)

Half of the sum of  $x$  and 6.2 is the same as 19.6 less than  $x$ .

Create and solve an equation to find the value of  $x$ .

Enter your equation and your solution in the space provided. Enter **only** your equation and your solution.

Equation:  $0.5(x + 6.2) = x - 19.6$

Solution:  $x = 45.4$

Item Information		
Answer:	See Image	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.EE.C.7.b	Solve linear equations with rational number coefficients, including equations with variables on both sides and whose solutions require expanding expressions using the distributive property and collecting like terms.
Evidence Statement:	8.EE.C.Int.1	Solve word problems leading to linear equations in one variable whose solutions require expanding expressions using the distributive property and collecting like terms. i) At least 80% of tasks should involve contextual real-world word problems (a noncontextual word problem could be "the sum of two times a number and 8 is 16").
Subclaim:	A – Major Content	The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.203	

Item Set 2 - Question 5 (TEI Inline Choice, Constructed Response)

Student A and Student B started collecting cans for recycling on the same day. Each student started with a certain number of cans, which is included in the total. The tables show the total number of cans each student had up to a certain day.

Student A		Student B	
Day	Total Number of Cans	Day	Total Number of Cans
10	116	15	205
40	207	32	240
70	295	57	289

**Part A**

Based on the tables, which student most likely collected the most cans by the 110th day?

Select from the drop-down menus to correctly complete each sentence.

most likely collected the most cans by the 110th day, which includes the cans the student started with. The best estimate of the total number of cans this student collected is .

**Part B**

- Using the table about Student B, create a model that can be used to estimate the number of cans,  $n$ , Student B will have on day  $d$ .
- Using your model, estimate how many cans Student B will have on the 80th day.

Enter your model and your answer in the space provided.

Item Information		
Answer:	Part A = See Image, Part B = See Sample Student Responses and Scoring Rubric	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.EE.C.8.c	Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
	8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two $(x, y)$ values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
Evidence Statement:	8.D.3	Micro-models: Autonomously apply a technique from pure mathematics to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense (e.g., profitably applying proportional relationships to a phenomenon that is obviously nonlinear or statistical in nature). Content Scope: Knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.
Subclaim:	D – Modeling and Application	The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course (or for more complex problems, knowledge and skills articulated in the standards for previous grades/courses), engaging particularly in the Modeling practice, and where helpful making sense of



	problems and persevering to solve them (MP. 1), reasoning abstractly and quantitatively (MP. 2), using appropriate tools strategically (MP.5), looking for and making use of structure (MP.7), and/or looking for and expressing regularity in repeated reasoning (MP.8).
P Value:	0.132

Scoring Rubric – Part A	
Points	Attributes
1	<b>Modeling Component:</b> <ul style="list-style-type: none"> <li>Student response is: Student A most likely collected the most cans by the 110<sup>th</sup> day, which includes the cans the student started with. The best estimate of the total number of cans this student collected is 415.</li> </ul>
0	Student response is incorrect or irrelevant.

Scoring Rubric – Part B	
Points	Attributes
2	Student response includes each of the following 2 elements. <ul style="list-style-type: none"> <li><b>Modeling component:</b> Valid model that can be used to estimate the number of cans, <math>n</math>, Student B will have on day <math>d</math>.</li> <li><b>Computation component:</b> Reasonable estimate based on the model of how many cans Student B will have on the 80<sup>th</sup> day.</li> </ul> <b>Note:</b> An expression equivalent to $n$ (e.g., $2d + 175$ ) may be acceptable as a model.
1	Student response includes 1 of the 2 elements.
0	Student response is incorrect or irrelevant.
Sample Student Response:	<b>Sample Solution 1:</b> A model for estimating the number of cans, $n$ , that Student B collects per day, $d$ , is: $n = 2d + 175$ To estimate how many cans Student B will have on the 80 <sup>th</sup> day, I substitute $d = 80$ into the model: $n = 2(80) + 175$ $n = 335$ I estimate that Student B will have a total of 335 cans on day 80.
Annotation for Sample Student Response:	<b>Solution 1, Score Point 2</b> The response receives full credit. It includes each of the 2 required elements. <b>Reasoning Component:</b> <ul style="list-style-type: none"> <li><b>Student Response:</b> A model for estimating the number of cans, <math>n</math>, that Student B collects per day, <math>d</math>, is: <math>n = 2d + 175</math>.               <ul style="list-style-type: none"> <li><b>Rationale for Score:</b> The student provides a valid model that can be used to estimate the number of cans, <math>n</math>, the student will have on day, <math>d</math> (<math>n = 2d + 175</math>). The model correctly represents how Student B's total number of cans collected, <math>n</math>, will change based on the number of days, <math>d</math>, that cans have been collected.</li> </ul> </li> </ul> <b>Computation Component:</b> <ul style="list-style-type: none"> <li><b>Student Response:</b> <math>n = 2(80) + 175</math>  <math>n = 160 + 175</math>  <math>n = 320 = 15</math></li> </ul>

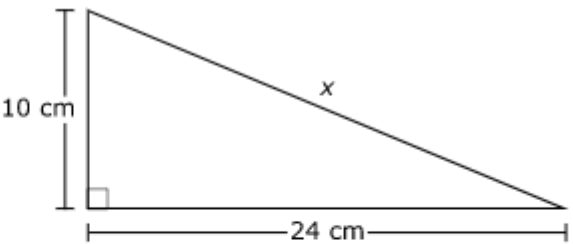
$n = 335$

- **Rationale for score:** The student provides a reasonable estimate for the total number of cans collected on day 80 using the student's model ( $n = 335$ ).

**Note:** Sample student responses are not representative of all correct answers for an item and are only provided as a guide to assist teachers with scoring.

Item Set 2 – Question 6 (Selected Response)

A right triangle is shown.



What is the value of  $x$ , in centimeters?

A. 17  
 B. 22  
 C. 26  
 D. 34

Item Information		
Answer:	C	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
Evidence Statement:	8.G.7-1	Apply the Pythagorean Theorem in a simple planar case. i) Tasks have "thin context" or no context. ii) Tasks require students to find one side of a right triangle in the plane, given the other two sides. iii) In 50% of tasks, the answer is a whole number and is to be given as a whole number. iv) In 50% of tasks, the answer is irrational and is to be given approximately to three decimal places. v) The testing interface can provide students with a calculation aid of the specified kind for these tasks.
Subclaim:	A – Major Content	The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.696	

Item Set 2 - Question 7 (Constructed Response)

Two companies rent boats by the hour. The total cost, in dollars,  $c$ , depends on the number of hours,  $h$ . The equations that represent the rental rates of both companies are shown.

Company A:  $c = 15h + 20$

Company B:  $c = 20h$

- A person rents a boat from Company A for  $h$  hours and realizes they would have paid the same amount if they had rented the boat from Company B. How many hours,  $h$ , did the person rent the boat? Explain or show each step of your work.
- Verify that your solution for  $h$  hours of renting the boat is the same cost,  $c$ , for each company.

Enter your answer and your explanations or steps in the space provided.

Item Information		
Answer:	See Sample Student Responses and Scoring Rubric	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.EE.C.8.c	Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
Evidence Statement:	8.C.4-1	Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4 = 5 + 7 = 12$ , even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions.
Subclaim:	C - Expressing Mathematical Reasoning	The student expresses grade/course-level appropriate mathematical reasoning by constructing viable arguments, critiquing the reasoning of others, and/or attending to precision when making mathematical statements.
P Value:	0.215	

Scoring Rubric	
Points	Attributes
3	Student response includes each of the following 3 elements. <ul style="list-style-type: none"> <li>• <b>Reasoning component:</b> Valid work or explanation of how to determine the number of hours needed to rent a boat from either company for the cost to be the same.</li> <li>• <b>Computation component:</b> Correct number of hours, 4.</li> <li>• <b>Reasoning component:</b> Valid work or explanation that the solution for <math>h</math> hours of renting a boat is the same cost, <math>c</math>, for each company.</li> </ul>
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 3 elements.
0	Student response is incorrect or irrelevant.
Sample Student Response:	The person rented the boat for 4 hours. To figure this out I took the 2 <sup>nd</sup> equation which was $c = 20h$ and plugged it into the first which was $c = 15h + 20$ to get this: $20h = 15h + 20$ $-15h -15h$ $5h = 20/5$ $h = 4$ So then to check I plugged 4 into the equations separately. Company A:

	$c = 20(4)$ $c = 80$ Company B: $c = 15(4) + 20$ $c = 60 + 20$ $c = 80$
Annotation for Sample Student Response:	<p><b>Score Point 3</b>  The response receives full credit. It includes each of the 3 required elements.</p> <p><b>Reasoning Component:</b></p> <ul style="list-style-type: none"> <li>• <b>Student Response:</b> I took the 2<sup>nd</sup> equation which was <math>c = 20h</math> and plugged it into the first which was <math>c = 15h + 20</math> to get this: <math>20h = 15h + 20</math>, <math>-15h -15h</math>, <math>5h = 20/5</math>, <math>h = 4</math>. <ul style="list-style-type: none"> <li>○ <b>Rationale for Score:</b> The student provides valid work to determine the number of hours needed to rent a boat from either company for the cost to be the same (<math>20h = 15h + 20</math>, <math>-15h -15h</math>, <math>5h = 20/5</math>, <math>h = 4</math>).</li> </ul> </li> </ul> <p><b>Computation Component:</b></p> <ul style="list-style-type: none"> <li>• <b>Student Response:</b> The person rented the boat for 4 hours <ul style="list-style-type: none"> <li>○ <b>Rationale for score:</b> The correct number of hours is provided (4 hours).</li> </ul> </li> </ul> <p><b>Reasoning Component:</b></p> <ul style="list-style-type: none"> <li>• <b>Student Response:</b> to check I plugged 4 into the equations separately. Company A: <math>c = 20(4)</math>, <math>c = 80</math>. Company B: <math>c = 15(4) + 20</math>, <math>c = 60 + 20</math>, <math>c = 80</math>. <ul style="list-style-type: none"> <li>○ <b>Rationale for score:</b> The student provides valid work to show that the hours determined (4), is the same cost for both Company A and Company B (Company A: <math>c = 20(4)</math>, <math>c = 80</math>. Company B: <math>c = 15(4) + 20</math>, <math>c = 60 + 20</math>, <math>c = 80</math>).</li> </ul> </li> </ul> <p><b>Note:</b> Sample student responses are not representative of all correct answers for an item and are only provided as a guide to assist teachers with scoring.</p>

Item Set 2 - Question 8 (Selected Response)

Two linear functions are described.

**Function Q**

$$y = 2x + 7$$

**Function R**

<b>x</b>	0	1
<b>y</b>	0	6

Which statement is true about the rate of change of Function Q?

- A. It is 3 times the rate of change of Function R.
- B. It is 2 times the rate of change of Function R.
- C. It is  $\frac{1}{2}$  the rate of change of Function R.
- D. It is  $\frac{1}{3}$  the rate of change of Function R.

Item Information		
Answer:	D	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greatest rate of change.
Evidence Statement:	8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greatest rate of change.</i> i) Tasks have “thin context” or no context. ii) The testing interface can provide students with a calculation aid of the specified kind for these tasks. iii) Equations can be presented in forms other than $y = mx + b$ , for example, $2x + 2y = 7$ .
Subclaim:	A – Major Content	The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.
P Value:	0.328	

Item Set 2 - Question 9 (Constructed Response)

Two snails climbed up a tree at a constant rate. A person measured and recorded their respective distances above the ground.

- Snail A was 12.5 inches above the ground at 10 minutes and 16 inches above the ground at 24 minutes.
- Snail B started at 3 inches above the ground and climbed 0.3 inch per minute.

The snails continued at the same speeds.

- Determine the amount of time, in minutes, it took for the two snails to be the same distance above the ground.
- Include an equation to represent each snail's distance above the ground,  $y$ , in terms of  $x$ , the minutes elapsed since the measurement started. Show your work or explain your answer.

Enter your answer, your equation, and your work or explanation in the space provided.

Item Information		
Answer:	See Sample Student Responses and Scoring Rubric	
Colorado Academic Standards (CAS) Evidence Outcomes:	8.EE.C.8.c	Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
Evidence Statement:	8.D.1	Solve multi-step contextual word problems with degree of difficulty appropriate to grade 8, requiring application of knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.
Subclaim:	D – Modeling and Application	The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course (or for more complex problems, knowledge and skills articulated in the standards for previous grades/courses), engaging particularly in the Modeling practice, and where helpful making sense of problems and persevering to solve them (MP. 1), reasoning abstractly and quantitatively (MP. 2), using appropriate tools strategically (MP.5), looking for and making use of structure (MP.7), and/or looking for and expressing regularity in repeated reasoning (MP.8).
P Value:	0.019	

Scoring Rubric	
Points	Attributes
3	Student response includes each of the following 3 elements. <ul style="list-style-type: none"> <li>• <b>Computation component:</b> Correct amount of time, in minutes, it takes for the two snails to be the same distance above the ground, 140.</li> <li>• <b>Modeling component:</b> Valid equation to represent each snail's distance above the ground, <math>y</math>, in terms of <math>x</math>, the minutes elapsed since the measurement started.</li> <li>• <b>Modeling component:</b> Valid work or explanation for how the equations and answer were determined.</li> </ul>
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 3 elements.
0	Student response is incorrect or irrelevant.
Sample Student Response:	Find rates Snail A = $\frac{3.5}{14} = 0.25$ inches per minute Snail B = 0.3 inches per minute Find starting distance

	<p>Snail A = <math>12.5 - (10 \times 0.25) = 10</math>          Snail B = 3          Equations          Snail A <math>y = 0.25x + 10</math>          Snail B <math>y = 0.3x + 3</math>          Solve as system of equations  <math>0.25x + 10 = 0.3x + 3</math>  <math>7 = 0.05x</math>  <math>x = 140</math>          So 140 minutes will have elapsed when they reach the same distance above ground.</p>
<p>Annotation for Sample Student Response:</p>	<p><b>Score Point 3</b>          The response receives full credit. It includes each of the 3 required elements.</p> <p><b>Computation Component:</b></p> <ul style="list-style-type: none"> <li>• <b>Student Response:</b> <math>x = 140</math>, 140 minutes will have elapsed when they reach the same distance above ground.             <ul style="list-style-type: none"> <li>○ <b>Rationale for Score:</b> The student provides a correct amount of time, in minutes, it takes for the two snails to be the same distance above the ground (140).</li> </ul> </li> </ul> <p><b>Modeling Component:</b></p> <ul style="list-style-type: none"> <li>• <b>Student Response:</b> Equations . . . Snail A <math>y = 0.25x + 10</math> . . . Snail B <math>y = 0.3x + 3</math> <ul style="list-style-type: none"> <li>○ <b>Rationale for score:</b> Valid linear equations are provided for snail A and Snail B that represent each snail’s distance above the ground, <math>y</math>, in terms of <math>x</math>, the minutes elapsed since the measurement started (Snail A <math>y = 0.25x + 10</math> . . . Snail B <math>y = 0.3x + 3</math>).</li> </ul> </li> </ul> <p><b>Modeling Component:</b></p> <ul style="list-style-type: none"> <li>• <b>Student Response:</b> Snail A = <math>\frac{3.5}{14} = 0.25</math> inches per minute, Snail B = 0.3 inches per minute          Snail A = <math>12.5 - (10 \times 0.25) = 10</math>, Snail B = 3, Snail A <math>y = 0.25x + 10</math>, Snail B <math>y = 0.3x + 3</math>          Solve as system of equations, <math>0.25x + 10 = 0.3x + 3</math>, <math>7 = 0.05x</math>, <math>x = 140</math> <ul style="list-style-type: none"> <li>○ <b>Rationale for score:</b> Valid work is provided to show how the equations and answer were determined (Snail A = <math>\frac{3.5}{14} = 0.25</math> inches per minute, Snail B = 0.3 inches per minute, Snail A = <math>12.5 - (10 \times 0.25) = 10</math>, Snail B = 3, Snail A <math>y = 0.25x + 10</math>, Snail B <math>y = 0.3x + 3</math>, <math>0.25x + 10 = 0.3x + 3</math>, <math>7 = 0.05x</math>, <math>x = 140</math>).</li> </ul> </li> </ul> <p><b>Note:</b> If the equations are incorrect, but the student solves for the amount of time it takes for the two snails to be at the same distance above the ground correctly, 1 computation point is earned.</p> <p><b>Note:</b> Sample student responses are not representative of all correct answers for an item and are only provided as a guide to assist teachers with scoring.</p>